The mathematical relation between collision risk and speed; a summary of findings based on scientific literature.

An exposé of scientific findings on road safety and speed written to accompany ETSC's PIN Flash 36 (2019) "Reducing speeding in Europe".

1. Introduction

There is hardly any subject that is researched more than the relation between road safety and speed. This is because we all know that higher speed implies a longer distance to come to a stop if anything goes wrong, and heavier collisions between road users with more energy to be dissipated and stronger forces to be dealt with. Speed can be measured, collisions can be counted, hence it has been a fruitful field of investigation and mathematical model fitting for many decades. One would expect that the relation between risk (e.g.. the number of road deaths per distance travelled, so as to correct for differences in travel) and speed would be established and well known by now. Unfortunately, this is not true.

There are almost as many results as there are researchers. For many policy makers, this forest of publications and reports doesn't exactly make it easy to decide which publication to take as a guide. This paper is meant to shed a light on the most important scientific publications, and help policy makers find their way in the context of management of speed on roads for which they are responsible. We start with a small choice of some of the issues that have led to confusion, and then proceed with a description of some of the more established results, their differences and properties. Finally we present a guide meant to help decide which compass to sail on.

1.1 Variations in definitions of speeds and collision risk

In this document we will represent "road danger" by collision risk. Collision risk equals the number of casualties (e.g. road deaths) per distance travelled. This definition is important, to ascertain that a change in travel of a person, or traffic volume on a road, is corrected for in the analyses.

There is massive research literature on the relation between speed and collision risk. Results show a wide variety of possible relations, largely because both speed and collision risks are also defined in various ways.

Speed is taken as (e.g.):

- speed of a single sampled vehicle
- speed, just before a collision, of a crashed vehicle, reconstructed from in depth crash research.
- locally observed mean speeds in a traffic sample on a specific road or set of roads
- speed limit of a specific road

The indicator associated with collision risk is taken as (e.g.)

- the number of collisions in the last three years (as stated by each driver)
- the number of crashed vehicles divided by the number of vehicles that did not crash (in a control group)
- the number of collisions in a specific period (before and after a change in speed, due to a change in speed limit, increased enforcement efforts etcetera) on a specific road or group of roads

And then, sometimes not the speeds, but the differences in speed between successive vehicles, or between an individual speed and the mean speed are brought into the discussion.

Ideally, the number of collisions is related to traffic volume, and subsequently the collision risk (i.e. number of crashed per distance travelled) should be analysed and compared.

1.2 Potential bias due to underreporting

Underreporting of road collisions is the phenomenon that not all collisions are reported by the police. This results in a so-called reporting rate (the proportion of collisions that are reported by the police) that is less than 1. Usually the reporting rate depends on several properties of the collision such as the severity, the number of casualties, the number of vehicles, the location etcetera. Research regarding the relation between speed and collision risk is based on numbers of observed (or self-reported) collisions, and as we know from e.g. ETSC's Pin Flash 35, even fatal collisions are not always reported, while less severe collision data are much less complete. Furthermore, as fatal collision data are often far too scarce to use as a basis for quantitative research, most research is based on serious injury collision data, and therefore likely to be subject to (gross) underreporting. Hence, all results may suffer from the influence of underreporting, which is worse for less severe collisions. As crash severity is likely to be correlated with speed, this implies that police collision reporting rates may vary with speed, i.e. reporting rates may be lower for lower speed. It is difficult to verify this assumption, but on the other hand, it is likely there is at least some truth in it, and it is therefore a good idea not to take the collision data at face value. As collision reporting rates may change over time, this may affect the results if reported collision data of several years are used in speed-risk analyses, especially data of injury collisions in year to year comparisons, that may be affected by changes in report rate. Hence, the uncertainty of collision rates based on police reports justifies a careful approach in data analyses in before-after studies.

The number of people killed, or injured, per fatal (or serious) collision constitutes another possible flaw in the reported data. These numbers are different in different countries, and this may well be caused by differences in underreporting of collisions with fewer victims. This effect may differ from country to country, and hence affect meta analyses regarding this variable. Hence, analyses of differences between countries of the relation between speed and the number of casualties per collision may be hindered by differences in report rate between countries.

1.3 Mathematical relations relating to risk and speed of traffic on a section of road

The results of speed-risk research invariably show that higher speeds (on a road) are associated with a higher collision risk (on that same road). The formulas used to quantify this relation come in different categories that are perhaps not always very well understood by readers and sometimes even authors. Two of the most important types of mathematical relations are:

- the power law: collision risk r is proportional to speed v raised to a certain power, or $r = C \cdot v^{\beta}$.
- an exponential law: collision risk r is proportional to an exponential function of speed v, or $r = C \cdot e^{\beta \cdot v}$.

Here, C and β are constants, where C often depends on speed limit, mean speed, the exact choice for the definition of r, etc. . The constant e represents Euler's constant, used in mathematics, and just as important as the well-known constant π, i.e. 3.141592653589794… . Euler's constant e approximately equals 2.718281828… . The function e^x has the interesting property that its derivative with respect to x also equals e^x.

An important property of the power law is that the elasticity between r and v is constant and equal to β. The elasticity is the factor between change in speed and change in risk. E.g. if $\beta=4$, this means that if speed increases by 10%, risk increases by 40%. This is true for the entire range of speeds for which the power law is (supposed to be) valid. The exponential function does not have that property of constant elasticity. On the contrary, if the relation between risk and speed follows an exponential function, elasticity changes with speed permanently. The higher the speed, the higher the elasticity.

These two function types are not easily distinguished in practice, based on data. One needs quite a number of data in a wide range of speeds, collected in well controlled experiments, to establish a specific power sufficiently accurately to be certain that the data follow a power law and not an exponential function, or the other way around.

When data are available and a decision has to be made upon the choice of the mathematical function, it is often a good idea to start with plotting the data in a graph with either linear or logarithmic axes., before deciding which mathematic form to choose for further analysis. If the risk values follow an exponential function of speed, the risk vs speed data will show as a straight line when risk is plotted on a logarithmic axis. If the risk values follow a power law, a straight line appears when the data are plotted in a double logarithmic figure (i.e. with both speed and risk axes being logarithmic). Excel facilitates these types of graphs; simply right-click on the axis of a figure in Excel and tick the logarithmic axis box.

1.4 Is it kinetic energy we deal with? And if not, what is the better alternative?

Many papers and reports regarding road safety and speed suggest it is the release of kinetic energy $E_k = ½mv^2$ that that explains the relationship: the higher the impact speed v of a vehicle or person with mass m, the more kinetic energy E_k is released, the more severe the consequences of a collision. Actually, this is not true. Even in the case of an almost perfectly elastic collision between two objects with people inside, where virtually all energy is conserved, people may be killed. The actual problem with road collisions is not the kinetic energy, but the change in momentum during the collision (i.e. the change in the product of speed and mass), Δmv, or thrust, that requires the brain and other body parts to decelerate very fast. Even if a body is perfectly protected from external damage, a very fast stop due to a collision may cause fatal injuries and death.

The thrust on a decelerating body changing from speed = v to zero speed in a short collision time Δt equals $mv=F\Delta t$, which means that the human body experiences a force F during a short time interval Δt. Hence the force F equals $mv/\Delta t$. Assuming the available distance s to come to a stop (the distance between the front of the car and the body of the car occupant, or the crumple zone) is approximately fixed, simple linear acceleration (i.e. a) equations yield: $s = v\Delta t + ½a(\Delta t)^2$ and $0=v + a\Delta t$; the latter implies $a=-v/\Delta t$, and substitution in the first equation yields $s = v\Delta t - ½v\Delta t = ½v\Delta t$. Hence $\Delta t = 2s/v$. Hence, as Δt decreases with increasing speed (the crashing vehicle has to come to a stop in a shorter time when speed is high), the force as a consequence of the collision is proportional to v^2.

We conclude that in a collision, the force on the body (and the brain), $F = mv/\Delta t = ½mv^2/s$, is proportional to v^2, just like the kinetic energy, $½mv^2$, is. Nevertheless the difference between this force and kinetic energy is very important. It explains that even if vehicles have a perfect crumple zone which absorbs virtually all energy, they still can't prevent occupants from being killed when speeds are high, especially when the vehicle is small and the crumple zone (and thus the Δt) is small as well. It is not the dissipation of energy that constitutes the danger, it is the deceleration, the thrust and the force on the body during the collision. If the car occupant is using a seat belt, this force is slightly decreased as the seat belt extends somewhat, thereby increasing the deceleration time, which is an important feature of the seat belt. The resulting force causes the brain to thrust forward inside the scull which is violently decelerated, which can cause fatal damaged. And of course, other parts of the body (such as the neck) may also be fatally damaged by these forces.

2. The most important findings in the scientific road safety research literature.

2.1 Single speed observations of drivers and their three year crash history in general.

Maycock et al (1998) researched drivers' observed speeds (on A roads, not excluding congestion) of drivers, and linked these to self-reported three year crash history of those drivers. They found (page 14) that the number of collisions increases with the 13th power of observed speed (measured at one of several specific sites). However, they do not give the standard deviation of their elasticity parameter (i.e. the value of the power, in this case 13), and further state that quite some unexplained variance remains, suggested to be attributable to other variables such as experience and annual distance travelled. They further give some unsupported statements:

1. They state: "*The usual figure quoted is that a 1 mph change in mean speed results in 5 per cent change in accidents - an effect size probably corresponding to an elasticity of between 1 and 2.*". This 1 mile/h – 5% risk change relation seems to refer to Finch' result (see section 2.4 of this document). This result implies an exponential relation between collision frequency and speed. An important property of such an exponential relation is that the elasticity varies with speed. When speeds equals 20 mile/h, a 1 mile/h increase is an increase of 5%, so the 5% risk change makes the elasticity equal to 1. When speed equals 40 mile/h, a 1 mile/h speed increase is an increase of 2.5%, hence the elasticity is 2. For a speed of 100 mile/h the elasticity would be 5. Maycock et al's conclusion (the part of the sentence following "probably") is highly remarkable: Finch's relation certainly cannot be associated with an elasticity between 1 and 2 at all speeds.

2. They state: "*…the fact that there is a strong association between speed and accidents does not necessarily mean that there is a causal link between the two - it seems more likely that the association arises from the fact that both speed and accidents are related in similar ways to the same variables - particularly age, experience and exposure.*"

Hence, they suggest that it is more likely that age, experience and exposure explain the high correlation between high speed and collision frequency, than that speed itself can be considered a causal factor. There is some sense in this: it seems trivial that speed behaviour is a consequence of human choices that, in turn, correlate with e.g. driver experience. But in that case it would be logical to model speeding behaviour as a function of human behaviour variables such as experience, and consider speed as the outcome that is more closely related to collision consequences than experience or age themselves. I would therefore rather suggest that age and experience influence speed behaviour, and speed influences collision frequency (although we shouldn't rule out the fact that inexperience, or high age, would add to risk within a group of drivers showing similar speed behaviour).

Given that in this study roads were binned into different mean-speed-regions, drivers who usually drove in congestion circumstances and drivers who usually used uncongested roads would end up in groups with different speeds.

Quimby et al. (1999) does the same type of research, but with free speed observations only. They find a less steep increase of collision frequency with speed, with an elasticity of 7.8.

Aarts and van Schagen (2006) show both results in one graph. It is unclear what explains this difference. Essentially, for roads with the same mean speeds, Quimby's sample essentially shows more collisions at low speeds (compared to Quimby's mean speed of 67km/h) than Maycocks sample (with an mean speed of 83km/h), while Quimby's increase with speed is less steep than Maycock's. Differences in driver travel behaviour may be substantial.

Below the two results with several combinations of linear and logarithmic scales for comparison.

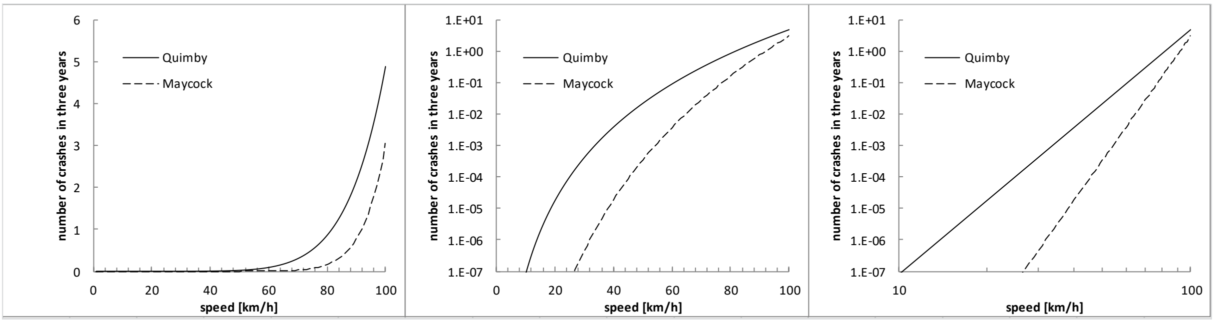


Figure 1. Results for Maycock's (v_{mean} = 83 km/h) and Quimby's (v_{mean} = 67 km/h) research with different combinations of linear and logarithmic scales, for easy comparison. The double logarithmic graph in the right panel shows the straight lines that appear as a consequence of the power laws.

Mark that both research results are based on a single speed observation for each driver, and a (self-reported) three year history of collisions related to this single observation. These collisions can have happened anywhere and they do not have even the slightest relation to this single speed observation on this specific road. The results of Maycock and Quimby refer to drivers, not roads and collision speeds. It is unclear to what extent their results can be used to relate speed on a road to the risk on that road. It could be that e.g. slow drivers (in free traffic conditions) usually do not drive on motorways, while they do drive on other (more unsafe roads) more often, thus explaining their higher numbers of collisions.

Aarts and van Schagen state that Maycock's as well as Quimby's suggestion that their results translate into an elasticity is valid for small speed differences only. They didn't recognise that the application of the power law exactly implies that these elasticities hold for the entire speed range.

<u>2.2 A case control study of collision vehicles' speed and collision risk, based on a comparison of reconstructed initial speeds of crashed vehicles compared to speeds of a control group of vehicles on the same road.</u>

Kloeden et al. (1997, 2001, 2002) researched the relation between serious collisions and speed by reconstruction of the speed prior to a collision, and comparing this speed to the mean speed of other vehicle's speeds (at the same location, time of day etcetera). This research strongly differs from the former group, as their research directly relates the reconstructed pre-crash speed to a factor that is proportional to the probability of a collision.

They find a result for 60km/h urban roads and for 80km/h (and higher) rural roads. For rural roads speed limits for the collision vehicles were 80km/h (17), 90km/h (2), 100km/h (43) and 110km/h (21); we can safely say these roads have a speed limit of approximately 100 km/h. Based on their data, and fitting collision vehicles with speeds at least equal to mean control group speed, (hence ignoring slow collision vehicle results), we find the following results:

- For urban 60km/h roads, collision risk increases with $e^{0.16v}$, suggesting a 16% increase in risk per km/h speed increase.
- For rural 80-110 km/h roads, collision risk increases with $e^{0.11v}$, suggesting an 11% increase in risk per km/h speed increase.

These results are not very accurate, because collision vehicle speeds were binned into speed groups with a 10km/h width, and the number of collision vehicles was rather low for most groups, and control group sizes were also rather small for higher speeds.

<u>2.3 Fatality risk for pedestrians in pedestrian-car collisions, as a function of passenger car impact speeds</u>

Kloeden's results, as they are based on pre-crash speeds and suggest an exponential relation between collision risk and speed, can be compared to results such as those of Rosén et al. (2011) who looked into the probability of a pedestrian to die in a collision with a passenger car with a specific impact speed. Their results suggest a 9% risk increase per km/h increase in impact speed for speeds up to 60km/h, 'as shown in Figure 2. The accuracy of this finding is again very poor, as it is based on few data, with a substantial variation. However, the logistic curve used by Rosén et al and others, is generally accepted, and implies an exponential relation between impact speed and risk for a large range of the speeds.

The research of Kloeden et al., and Rosén et al. is based on reported collisions, and mixes fatal and serious collisions. It is therefore likely that less severe (lower speed) collisions are missing in the dataset, whereas extremely severe collisions are more likely to be included in the dataset. If this is indeed the case, Kloeden's and Rosén's results are tilted towards higher exponential parameter values.

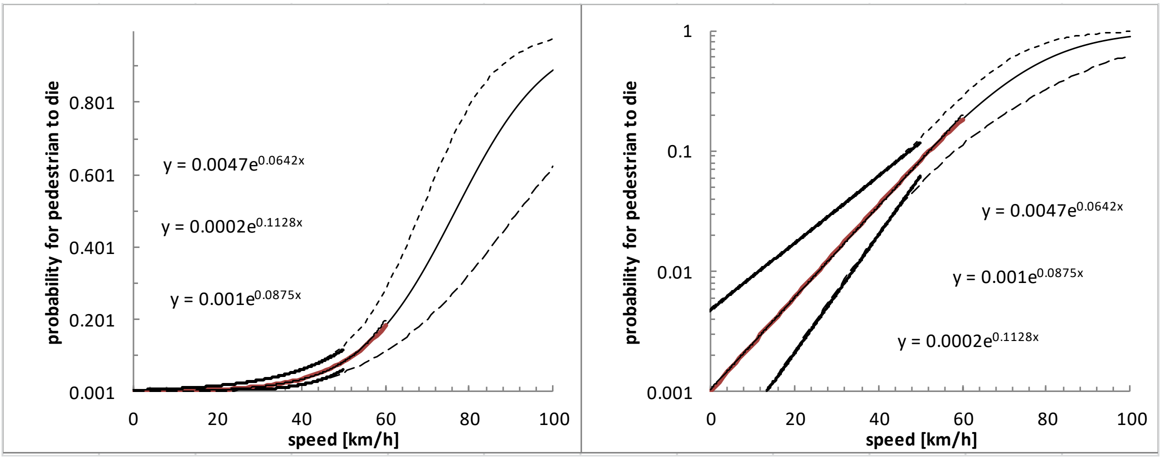


Figure 2. Results of Rosén et al. for the relation between the probability for a pedestrian to be killed by a car and the impact speed of that car (thin solid line). Dotted lines denote uncertainty margins, thick solid lines denote exponential trend lines.

<u>2.4 The relation between mean speed and collision risk before and after a speed limit change</u>

Finch et al. (1994) analysed observed mean speed data and collision frequency data before and after speed limit changes in several countries. They found an *exponential* relation between collision risk and speed, expressed as a *linear* relation between relative collision risk *change* and absolute speed change. These two are equivalent, because an exponential relation between risk and speed ($r = C \cdot e^{\beta \cdot v}$) implies that when v increases by 1 km/h, i.e. $v_2 - v_1$ =1km/h, the corresponding risks have a fixed ratio:

$r_2/r_1 = e^{\beta}$ and the relative risk change $(r_2 - r_1)/r_1 = r_2/r_1 - 1$ is a value that is more or less constant for small values of β. Unfortunately, this relation cannot be extended to larger speed differences without accepting that the risk ratio changes with the size of the speed difference. E.g. for $v_2 - v_1$ = 2km/h, $r_1/r_2 = e^{2\beta}$. For small values of β the relation between relative risk change and speed change is “almost linear”, e.g. for β =0.03 [h/km], $e^{\beta} - 1 = e^{0.03} - 1 \approx 0.030$, and $e^{2\beta} - 1 = e^{0.06} - 1 \approx 0.062$, hence it is slightly larger than twice the value of e^{β}. As a consequence, the larger the difference in speed, the more the change in risk ratio will deviate from the presumed linear relation.

Finch et al did not point this out. He indeed observed that for large changes in speed, his results seem to deviate from the desired linear relation, as could have been expected because of the exponential relation as a starting point. Finch subsequently treats the result as if the fit needs a correction term in the mathematical expression for risk. This correction term evidently is not needed. In fact it is superfluous, as his simple exponential result explains his data nicely. This is illustrated in Figure 3, where Finch’s results (taken from the graph in the report) are plotted and fitted with an exponential function. The values of “percent change in accident” i.e. $(r_2 - r_1)/r_1$ are rewritten as a relative risk, i.e. r_1/r_2, and the speed values (in mile/h) are transferred to values in km/h.

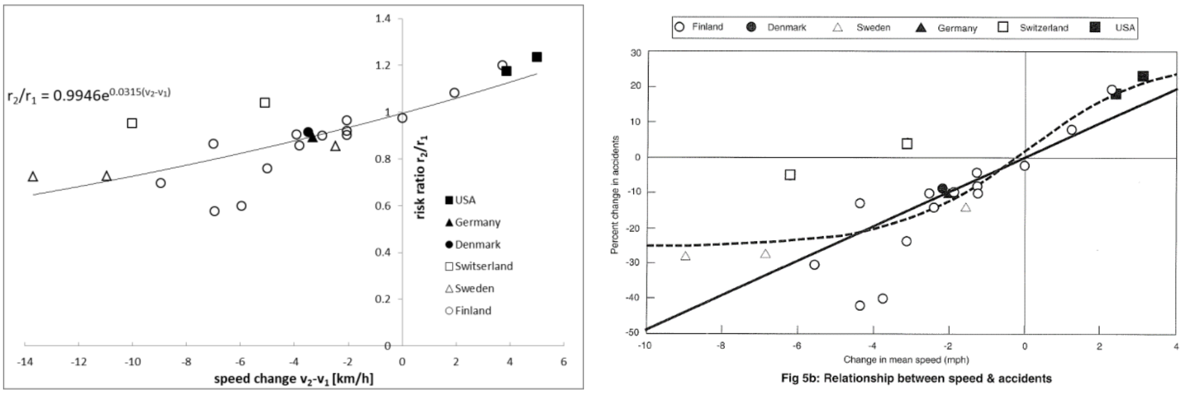


Figure 3. Data from Finch et al., plotted and fitted with an exponential function (left panel) and with a straight line, as well as a straight line with a correction function (right panel; copied and pasted from Finch’s paper). The formula in the left panel represents the exponential trend line, as obtained with Excel. Different symbols correspond to different countries, in agreement with Finch’s data.

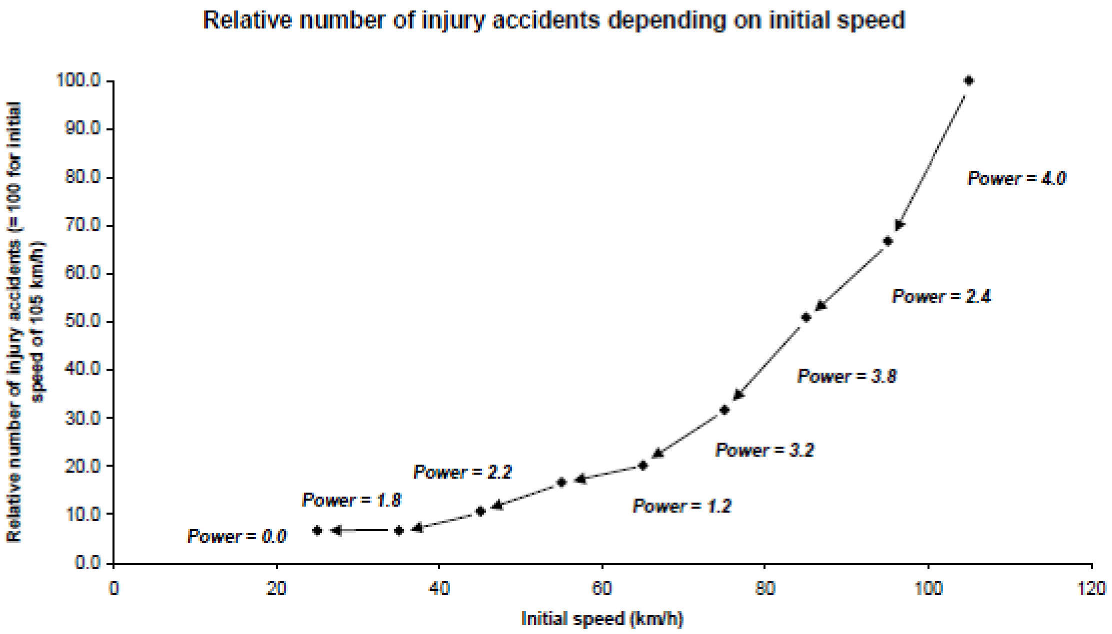
2.5 Nilsson’s power model and Elvik’s evaluations

In the eighties of the 20th century, Nilsson (1982) postulated his power laws regarding the relation between speed and risk. He later supported his power laws with experimental evidence and increased the complexity of the model by distinguishing fatal collisions and fatalities, and serious injury collisions and serious injuries, etc.(Nilsson, 2004). At the same time, also Elvik et al. (2004) revisited the original power laws, presenting a meta-analysis with a legible overview of relevant papers regarding before-after studies of speed limit changes or enhanced enforcement policies.

Nilsson’s original postulate suggested a fourth power relation between road death risk and speed and a third power for serious injuries. In his 2004 publication he gave ranges of powers for fatalities and injuries. Elvik et al. showed that the powers need adjustment depending on collision severity, and they also found different results for different methodological approaches, with sometimes large standard errors, which give room for multiple interpretations.

Elvik et al. also considered other models, e.g. they presented their data in the same variable space as Finch and colleagues did, i.e. with a change in speed vs a risk ratio. There, just like Finch et al., they suggest a linear relation between relative risk change and speed change, which actually yields an exponential relation between risk and speed (c.f. Figure 20 of the report). Further, they recognize the fact that for very low speeds (say 5km/h) a power law such as Nilsson’s fourth power law is unlikely to be valid. Hence, they present the result of an analysis of a subset of their data that allowed for stratification by initial speed. This very interesting analysis shows that the actual power of speed is higher for high speed roads than for lower speed roads.

This is very interesting indeed, because their power laws with different powers for different speed regimes actually yields an exponential relation! Figure 4 shows a cut and paste copy of Figure 21 of Elvik et al., 2004.



Source: TØI report 740/2004

Figure 4. A cut and paste copy of Figure 21 in Elvik et al., 2004.

The data, as presented in Figure 4, were taken and plotted as if representing an exponential function (i.e. with a logarithmic vertical scale, so as to show a straight line in case the data are indeed representing an exponential function). The result is shown in Figure 5, where, indeed, an almost straight line appears. This line represents an exponential function of relative risk as a function of speed change, with an exponential parameter of 0.037/(km/h). This is very similar to Finch et al. results described in Paragraph 2.4.

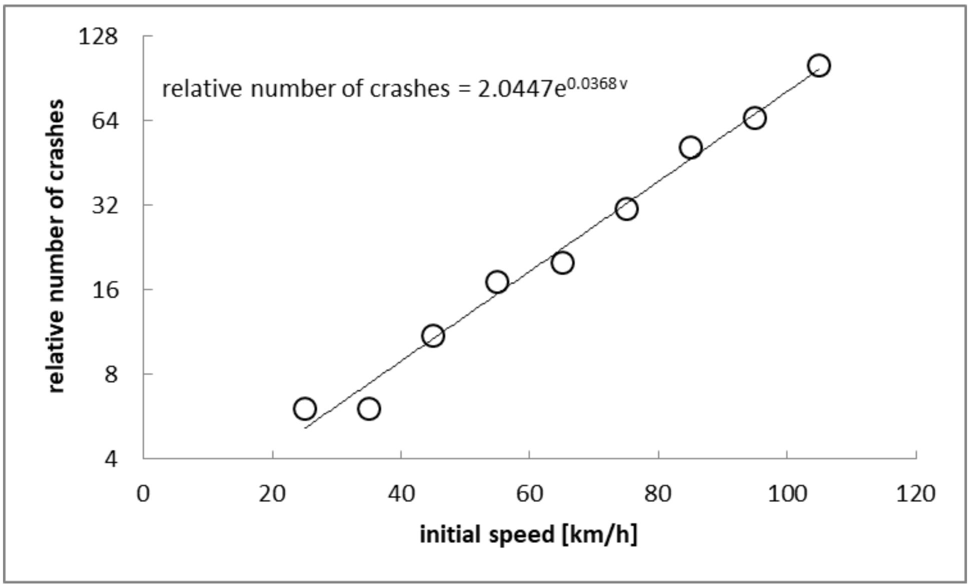


Figure 5 Data from Figure 4, plotted on a loglinear scale so as to illustrate the exponential relation between collision risk and speed. This relation is represented by the solid line, as produced by Excels trend line function. Mark that the constant 2.044 is meaningless, as the vertical axis is scaled so as to have the data point at v=100km/h correspond to 100.

Elvik et al. conclude that these results do not justify an alternative model, because they suspect the results to be insufficiently accurate. However, this argument also holds for the proposed power law, as the power they propose is supposed to be valid for all speeds, which clearly is not transparently argued. We may therefore conclude that the available results do not conclusively prove that Nilsson's power laws are more valid than Finch's exponential law.

In (Elvik, R. (2009). The Power Model of the relationship between speed and road safety: update and new analyses. TØI Report 1034/2009. Oslo, Institute of Transport Economics TØI.), Elvik comments on a 2007 publication by stating that three re-analyses of the original study have been made, e.g. by Cameron and Elvik, (M.H. Cameron, R. Elvik, Accident Analysis and Prevention 42 (2010) 1908–1915), all concluding that the effect of a given relative change in speed depends on the initial level of speed. They establish that this is not consistent with the power model. The problem is solved by applying two different sets of power models, one for rural and highway high speed roads and for urban and residential low speed roads.

Several years later, Elvik (2013) concludes that an exponential function better describes the relation between collision risk and speed, with an exponent of approximately 0.034/(km/h) for injuries, which is much alike Finch's result and the result depicted in Figure 5. For fatal collisions he denotes an exponent of approximately 0.069 per km/h, i.e. approximately twice the parameter for injuries, and more like the parameter derived from Rosén's data (cf Figure 2). Elvik's results for fatalities suggest that the data tend to deviate from the exponential fit for high speeds, which is also in agreement with Rosén's result (c.f. paragraph 2.3).

Finally, Elvik et al. (2019) conclude on the basis of an updated review of relevant studies that the best current estimates of the speed coefficient in the exponential model are 0.08/(km/h) for deaths and 0.06/(km/h) for injury collisions. The value for deaths is somewhat higher than in previous meta-analyses, which the authors find understandable, but the value for injury collisions is near double the values from previous studies and the authors offer no explanation for that. The value of 0.08/(km/h) implies that a speed increase of 1 km/h yield a risk increase of $e^{0.08}$ = 8.3%.

3. Can Nilsson's and Finch's results be valid at the same time?

Let's assume for a moment that Finch's results (and Elvik's eventual 2013 results) are valid, and that collision risk r increases with mean speed v according to $r = a\, e^{-0.03v}$, or perhaps with a slightly different exponent, and a being any suitable constant. This would mean that a change in mean speed for a specific road very likely resembles a power law, with a power that depends on the initial mean speed, and approximately equal to 4 for mean speeds between 80km/h and 100km/h, but lower for lower mean speeds (and higher if Elvik's exponent derived for fatal collisions in 2019 is correct).

There is no obvious reason why this should not be true. The available evidence for Nilsson's law leaves room for deviations or different powers, as data tend to show as rather widely distributed clouds in the speed change vs collision risk change diagrams.

But Nilsson's law and Finch's law cannot be true at the same time for a large range of speed.

4. Conclusion

Elvik's 2013 paper suggests that the relation between collision risk and speed is exponential, and not a power law. Of course, within a small range of speeds and speed differences, the data may well display a power law relation such as described by Nilsson. However, the power parameter would depend on speed, and thus have only limited validity. Earlier publications, e.g. Finch et al, and Elvik (2004) indicate this exponential relation between risk and speed as well (see figure 3 and figure 5 of this document), although unfortunately the authors themselves didn't conclude this at that time.

For injuries, Elvik's 2004-results suggest that risk is proportional to $e^{0.034v}$, with v in km/h and the parameter 0.034 in 1/(km/h). This is in good agreement with Finch's result, who's data give rise to an exponential relation with a parameter of 0.037/(km/h). Finch also analysed non-fatal collisions mainly. For road deaths, Elvik found in 2013 that risk of fatal collisions is proportional to $e^{0.069v}$. This is somewhat similar to Rosen's result for the probability of a pedestrian to be killed in a collision with a car with impact speed v. Apparently these values are likely to be of the right order of magnitude. In 2019 Elvik and others found that risk of a death is proportional to $e^{0.08v}$; this being higher than $e^{0.069v}$ is plausible in that the number of deaths per fatal collision is likely to increase with speed, but this is not a complete explanation of the difference

It is our view that Elvik's 2019 paper currently constitutes the most accurate description of the relation between the risk of death in a collision on a road and the mean speed on a road. It is not entirely contradictory to Nilsson's and Elvik's original power laws, but these power laws have a much more limited range of validity.

Those who would still want to continue to use Nilsson's laws are advised to derive the power they should apply from Elvik's results. Essentially, for a road where traffic has a mean certain speed, the power law parameter to apply when this mean speed is changed, depends on the initial speed. This power can be derived, according to the following mathematical reasoning:

When risk r (either road death risk or injury risk) relates to speed v according to $r = C.e^{\beta v}$ and speed is changed from v_1 to v_2, (i.e. mean speed changes by a proportion $(v_2-v_1)/v_1$), then risk will change from $Ce^{\beta v_1}$ to $Ce^{\beta v_2}$ and the proportion of risk change will be (independent of the constant C): $(r_2-r_1)/r_1 = (e^{\beta v_2} - e^{\beta v_1})/e^{\beta v_1} = e^{\beta(v_2-v_1)} - 1$.

The appropriate power, necessary to describe the relation between the relative change in speed and the relative change in risk, can be found by filling in the values, e.g. when mean speed changes from 100km/h to 90 km/h and $\beta = 0.08$h/km, we find:

$(v_2-v_1)/v_1 = (90-100)/100 = -0.1 = -10\%$

and

$(r_2-r_1)/r_1 = e^{\beta(v_2-v_1)} - 1 = e^{-0.8} -1 = 0.45 – 1 = -0.55 = -55\%$.

This suggests a power of 5.5, *based on the exponential relation, and valid for this speed range only*!

5. Acknowledgements

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